

Geometer's Skethchpad 8th Grade Guide to Learning Geometry



This Guide Belongs to:

Date: _____

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***a story can be added or one could choose to use the activities alone and add their own fun and games to make it interesting!*

Getting Started with Sketchpad

What is Sketchpad?

Sketchpad is a computer program specially designed for drawing and manipulating geometric figures. Using Sketchpad is a fun way to learn about and experiment with geometry.

Activities in this booklet

There are 14 activities in this booklet covering a wide-range of geometric concepts. You will work with a partner to complete each activity. You and your partner will share unique jobs throughout these activities. For each activity you will be either a **Mouse Operator** or an **Activity Recorder**. At the end of each activity you will change jobs with your partner.

A lot of geometry terms are **not** described in this booklet. When you see words with which you are not familiar, you should look in your textbook to find out more about the subject. Words that you see in **bold** may be new words for you.

Using Sketchpad

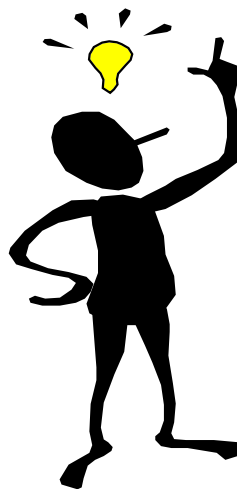
In this section, you will learn some of the basics of using Sketchpad. You will find other information about using Sketchpad in other sections of this booklet. All the information about Sketchpad you need to do the activities can be found in this booklet.

Creating a new sketch

Create a new sketch by clicking on the File menu and selecting New Sketch. A blank sketchpad will appear.

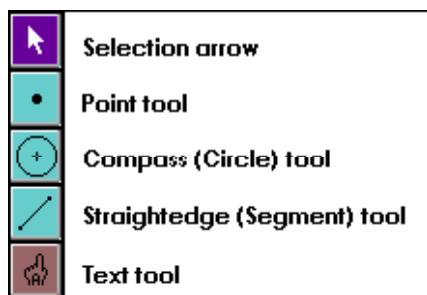
Saving a sketch

During these activities, you may want to save your work to a disk. To save a sketch, click on the File menu and select Save. You will be asked to type in a name for your file. (The name must be 8 characters or less.)



IMPORTANT: Before beginning, turn on Auto Show Labels for points and circles. To do this, click on the Display menu and select preferences. Click on the boxes beside Points and Circles if they are not already selected. Also, while in the Preferences dialog box, change the Distance Unit preference to centimeters (cm) and set the precisions to tenths.

The toolbar



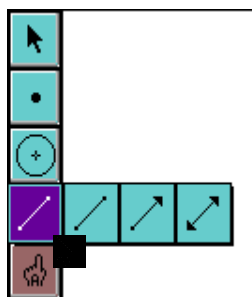
• **Selection arrow**—use this tool to click on objects to move them or resize them. Also use this tool to select (or highlight) objects.

Important: To select one or more objects at the same time, hold down the Shift key and click on all of the objects you want selected.

• **Point tool**—use to place points on your sketch.

• **Compass tool**—use to draw circles.

• **Straightedge tool**—use to draw segments, rays, and lines (you will learn more about these in Activity 1). Click and hold down the Straightedge tool to see the options for segments, rays, or lines.



• **Text tool**—use to change object labels. Click on an object to display or hide its label. Double click on a label to change it.

Clicking and dragging

Something you will be doing a lot in Sketchpad is “clicking and dragging” objects. **Click** means to position the pointer over an object and press the mouse button (and not let go) to select it. **Drag** means to then move the mouse to move the object on the sketchpad.

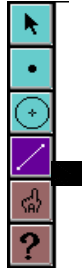
Investigating Points & Lines

How to draw a point

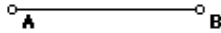
1. Select the Point tool by clicking on it in the toolbar.
2. Position the cursor where you want to place the point. Click. The point appears on your sketchpad.

How to draw a line segment

1. Select the Segment tool by clicking on it in the toolbar.



2. Click and drag the mouse across the page to draw a line segment.

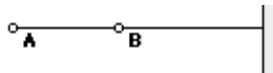


How to draw a ray

1. Click and hold the Segment tool on the toolbar. Select the Ray tool.



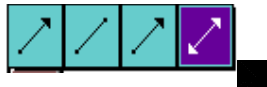
2. Click and drag the mouse across the page to draw a ray.



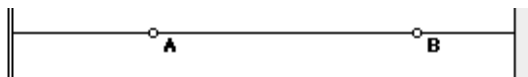
Notice how the ray starts at point A, passes through point B and extends off the sketchpad on one side.

How to draw a line

1. Click and hold the Segment tool on the toolbar. Select the Line tool.



2. Click and drag the mouse across the page to draw a line.



Notice that the line extends off the page on both sides.

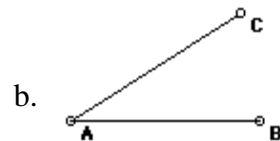
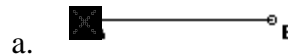
Investigating Angles

How to draw an angle

1. Select the Segment Tool by clicking on it in the toolbar.
2. Click and drag the mouse across the page to draw a line segment.

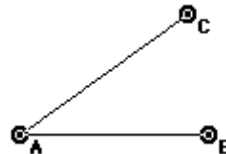


3. While the segment tool is still selected, move the mouse cursor over point A, then click the mouse button and drag to make a new line segment above the first line segment (like Figure b. below).



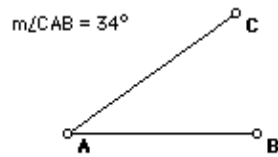
How to measure an angle

1. Click the selection arrow in the toolbar. Click on one of the endpoints (for example C or B below), then hold the Shift key down and click on the **vertex** and then the other endpoint. (You must select the vertex second.)



2. Click on the Measure menu and select Angle.

You should now see the measure of the angle in the upper left-hand corner of your sketch. Notice the way the angle is labeled. The letters should be in the same order as you selected them.



Investigating the Pythagorean Theorem

A special relationship exists between the sides of a right triangle. In Activity 11, you will investigate this important relationship. Before you get started, though, you need to know a couple of terms used when talking about a right triangle:

- the side opposite the right angle is called the **hypotenuse**
- the other two sides are called **legs**.

• Activity 1 •

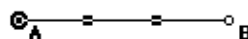
Swap jobs with your partner. Create a new sketch.

1. Follow the steps below to create a right triangle.

Select the Segment tool from the toolbar. Hold down the Shift key (this will keep the line horizontal) and drag the mouse to create a line segment about the length of AB below.

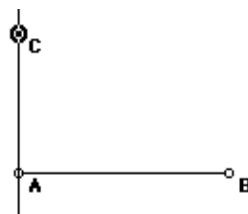


Select the left endpoint and the line segment.



Click the Construct menu and select Perpendicular line.

Place a point on the perpendicular line a little above the original endpoint (like point C below).

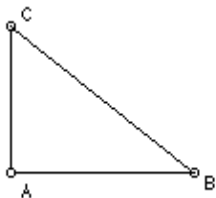


Select the new point you just added and **both** endpoints of the original segment.

Click the Construct menu and select Segment.

Click the perpendicular line, then click the Display menu and select Hide Line.

This will complete your triangle. It should look similar to this one:



2. Verify that this is a right triangle by measuring the right angle.

What measure did you get?

3. Measure the two legs and the hypotenuse. Write your measures below.

• leg 1 _____

• leg 2 _____

• hypotenuse _____

3. Drag any of the points of the triangle in any direction.

What happens?

4. Use your calculator to square the length of the legs and the hypotenuse. Write the squared measures in the blanks below.

• leg 1² _____

• leg 2² _____

• hypotenuse² _____

5. Add the two squared measures for the legs and compare this to the squared measure of the hypotenuse.

What do you notice?

6. Drag any point of the triangle to make it larger.

7. Repeat steps 1 and 2 above. Fill in your answers below.

• leg 1 _____

• leg 2 _____

• hypotenuse _____

• leg 1² _____

• leg 2² _____

• hypotenuse² _____

8. Again, add the squared measures of the legs and compare that to the squared measure of the hypotenuse.

What do you notice?

Do you think all right triangles would give you the same results? Why or why not?

Will you get the same results with a triangle that is not a right triangle? Why or why not?

9. Explain the relationship between the 2 legs and the hypotenuse of a right triangle.

10. The Pythagorean Theorem is:

$$c^2 = a^2 + b^2$$

Does this theorem resemble your answer for number 9?

END OF ACTIVITY 1

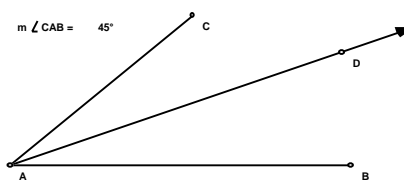
Angle Bisectors

• Activity 2 •

Swap jobs with your partner. Create a new sketch.

1. Construct a $45\frac{1}{2}$ angle. Be sure to measure the angle.
2. Click on the three points (the angle side, the vertex, and the other angle side) just like you are measuring the angle.
3. Click on the **Construct** menu and select **Angle Bisector**.
4. Place and label a point on the ray.

Your sketch should look similar to this:



5. Measure each of the new smaller angles.

What is the measure of each angle?

Sum the two small angles. What do they add up to?

6. Drag the endpoint of segment CA to the left.

Sum the two smaller angles again. What do they add up to?

Ray AD **bisects** angle CAB. What do you think is a good definition of an angle bisector?

END OF ACTIVITY 2

Measuring Perimeter and Area

In this section, you will look at different measures of other two-dimensional geometric figures.

• Activity 3 •

Swap jobs with your partner.

1. Open the sketch named **Activ3.gsp**.
2. Measure the length of each side of quadrilateral CDBA.

w = _____

x = _____

y = _____

z = _____

3. What is the **perimeter** of this figure?

4. Drag point B to resize the figure. Write your new lengths for the sides.

w = _____

x = _____

y = _____

z = _____

5. What is the new perimeter?

6. Check your answers by using the measure menu to measure the perimeter.

Select all four points of the figure (hold down the Shift key and click on all four).

Click on the Construct menu and select Polygon Interior.

Click on the Measure menu and select Perimeter.

Does Sketchpad give you the same answers you came up with? If not, list the answers Sketchpad gave you.

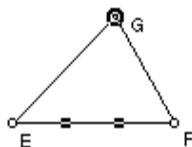
7. Give your definition of perimeter.

END OF ACTIVITY 3

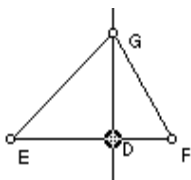
How to measure the height of a triangle

You will need to measure the **height**, or altitude, of a triangle to be able to calculate its area. Here's how you do it.

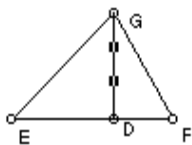
1. Select the base of the angle and the vertex opposite the base.



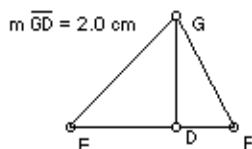
2. Construct a perpendicular line by clicking on the construct menu and choosing perpendicular line.
3. Place a point at the intersection of the base and the perpendicular line.



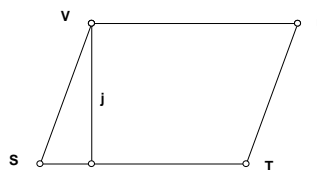
4. Click on the perpendicular line and choose Hide Line from the Display menu.
5. Select the new point on the base and the vertex opposite the base. (Remember to hold down the shift key and click on both points.)
6. Click on the Construct menu and select Segment.



7. While the new line segment is still selected (as shown above) click on the Measure menu and select Length. The length of that segment is also the **height** of the triangle.



Parallelograms also have height, or altitude. Look at segment *j* below.



You need the height of a parallelogram to calculate its area.

You also need the base.

• Activity 4 •

Swap jobs with your partner.

1. Open the sketch named **Activ 4.gsp**.
2. Measure the Base and Height of figure 1.

Base = _____

Height = _____

FORMULA:

*Area of rectangle = base x hieght
or
length x width*

3. Using a calculator (or by hand), calculate the **area** of the rectangle in figure 1. The formula is at the left.

What unit of measure did you use to state your answer? Why?

4. Measure the base and height of the triangle in figure 2.

Base = _____

Height = _____

5. Using a calculator (or by hand) calculate the area of the triangle. The formula is at the right.

FORMULA:

*Area of triangle = $1/2 hb$
 h is the height
 b is the base*

6. Measure the base and height of the parallelogram in figure 3.

Base = _____

Height = _____

7. Using a calculator (or by hand) calculate the area of the parallelogram. The formula is at the right.

FORMULA:

Area of parallelogram = hb

*h is the height
 b is the base*

Take the following steps to check your answers using the Measure menu.

8. Select the 4 vertices of the quadrilateral in Figure 1 in this order: **A, B, D, then C**. Click on the Construct menu and select Polygon Interior.

9. Click on the Measure menu and select Area.

What is the measure?

10. Select the 3 vertices of the triangle in Figure 2. Click on the Construct menu and select Polygon Interior.

11. Click on the Measure menu and select Area.

What is the measure?

12. Select the 4 vertices of the quadrilateral in Figure 3 in this order:

S, T, U, then V. Click on the Construct menu and select Polygon Interior.

13. Click on the Measure menu and select Area.

What is the measure?

14. Using the information you already know about area, determine the area of the polygon in Figure 4? Record your answer below.

15. What is your definition of area?

END OF ACTIVITY 4

Activity 5

Swap jobs with your partner.

1. Open the sketch called **Activ5**. Answer the questions for both Figures 1 and 2.

Area of Square in Figure 1 = _____

Area of Circle in Figure 1 = _____

Circumference of Circle in Figure 1 = _____

Area of Yellow Section in Figure 2 = _____

Circumference of Circle in Figure 2 = _____

Formula for Area of a square

$$= S^2$$

S = Side

Formula for area of a

Circle =

Pi x r²

r = radius

2. Look at Figure 1 and answer the following:

If you only knew the area of the circle, would you be able to answer the three above questions about Figure 1? How?

3. Look at Figure 2 and answer the following:

If you didn't know the radius of the circle, could you answer the last two questions? Why or why not?

END OF ACTIVITY 5

Activity 6

Swap jobs with your partner.

1. Open the sketch called **Activ6.gsp**.
2. Measure the **area** of the blue triangle using the Measure menu. Remember to construct the Polygon Interior first.

What is the area?

3. Drag point E back and forth.

Does the area of the triangle change?

2. Look at Figure 1 and answer the following questions:

What is the area of the square?

What is the area of the circle?

What is the circumference of the circle?

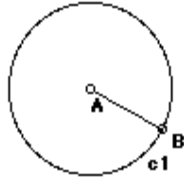
If you did not know the length of one of the sides of the square, what information would you want to help you

END OF ACTIVITY 6

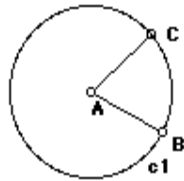
• Activity 7 •

Swap jobs with your partner. Create a New Sketch.

1. Construct a small circle.
2. Construct the circle's radius, from the center point to the control point. Your sketch should look similar to this:



3. Add another point on the circle, away from the control point.
4. Construct another segment to connect this new point with the center point. Your sketch should look similar to this:



Try to drag the new point. What happens?

In Sketchpad, you can only drag the center point or control point to change the size of a circle.

5. Measure the length of each segment.

What do you notice about the measures?

6. Drag the control point to change the size of the circle.

What happened to the measures?

Is the second segment you created also a radius? Why or why not?

Give your definition of a **radius** of a circle.

7. Construct a chord on your circle (but do not use the control point as one of the endpoints). Refer back to page 18 for a reminder.
8. Click on one endpoint of the chord and drag it. Move the point to different places along the circle.

What happens when you drag the point of the chord?

When you move the endpoint, does the segment remain a chord?
Why or why not?

END OF ACTIVITY 7.

Leave sketch open for Activity 8.

Activity 8•

The radius is an important part of a circle. When you know the radius of a circle, you can also determine its **diameter**, **circumference**, and **area**. In this activity you will measure a circle's radius and then use formulas to find the circle's diameter, circumference, and area.

Swap jobs with your partner. Make sure you have the sketch from **Activity 7** open.

1. Using the measure of the radius in your circle and the appropriate formulas find the following:

- the diameter $2r$
- the circumference $2\pi r$
- the area πr^2

Radius = _____

Diameter = _____

Circumference = _____

Area = _____ (your answer should be
in square units)

2. Click and drag the control point so that the radius of your circle is 3 cm. Determine the following:

Radius = _____

Diameter = _____

Circumference = _____

Area = _____

Use the Measure menu to check your answers.

1. Click anywhere along the circle except on a point.
2. Click on the Construct menu and select Circle Interior.
3. While the interior of the circle is highlighted, click on the Measure menu and select Radius.
4. Click the Measure menu again and select Circumference.
5. Click the Measure menu again and select Area.



To double check the diameter, just multiply the radius measure by 2.

6. What measures did Sketchpad come up with?

Radius = _____

Diameter = _____

Circumference = _____

Area = _____

END OF ACTIVITY 8

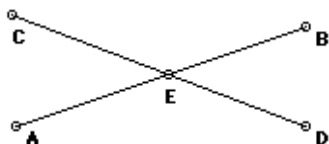
Vertical Angles

• Activity 9 •

Swap jobs with your partner. Create a new sketch.

1. Click on the Line tool in the toolbar and construct a line segment.
2. Construct a second line segment that intersects the first at about the midpoint.
3. Place a point at the **intersection** of the two segments.

Your sketch should look similar to this:



How many angles do you see?

What kind of angles do you see?

Name a pair of vertical angles.

2. Measure two vertical angles . What do you observe?

3. Click the endpoint of either one of the segments and drag it in either direction.

What happens to the angle measures?

Explain what makes two angles vertical angles.

END OF ACTIVITY 9

Investigating Special Lines and Angles

Using the Construct Menu to draw lines

In Sketchpad, you can use shortcuts from the Construct Menu to help you draw figures. In the activities in this section and later activities, you will use the Construct Menu to help you construct many of your sketches.

How to construct a segment

1. Select two points that you want to be the endpoints of a segment. Remember to hold the shift key and click on both of them.



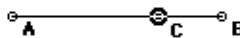
2. Click on the Construct Menu and select Segment. You should now see a line segment with the two selected points as the endpoints.



You could quickly construct a triangle by selecting three points and choosing Segment from the Construct Menu. Sketchpad draws segments between each of the points to make the triangle.

How to construct a perpendicular line

1. Construct a segment.
2. Place a point on the line.
 - a. Click the Point tool on the toolbar.
 - b. Move the mouse over the line segment and click where you would like to place the point.

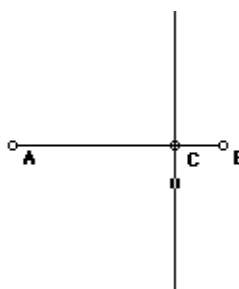


- Click the selection arrow in the toolbar. Select the new point and the line segment by holding down the shift key and clicking both the point and the line segment.



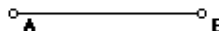
- Click on the Construct Menu and select Perpendicular Line.

You should now see a new line running through the point on the line segment. This new line is a perpendicular line.

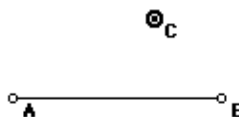


How to construct a parallel line

- Construct a line segment.

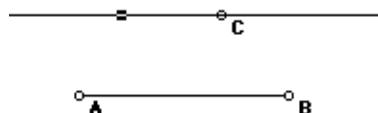


- Place a point above or below the line segment.



- Select both the point and the line segment.

- Click on the Construct Menu and select Parallel Line. You should see a new line above the first line segment. This is a parallel line.



• Activity 10 •

Swap jobs with your partner. Create a new sketch.

1. Construct a line segment. Use the method described in this section (using the Construct menu).

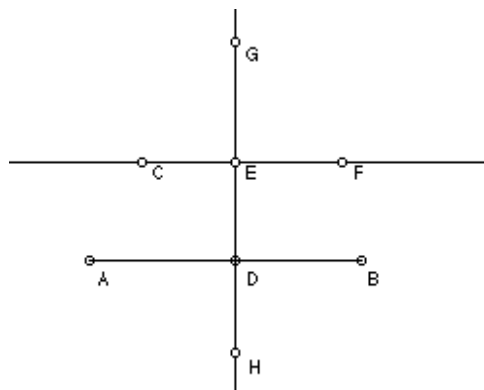
Label the line segment **j**. Click on the Text tool and click once on the line segment. Then double-click on the label. Enter the letter **j** and press return

2. Construct a line parallel to line segment **j**. Remember to place a point above line segment **j**.
3. Place a point on segment **j** (point **D** in the sketch below—remember, your letter may be different).

Construct a line perpendicular to segment **j** that runs through the point.

4. Place a point at the **intersection** of the parallel and perpendicular lines you created (point **E** below).
5. Add points to the right of the intersection (**F**) and just above the intersection (**G**). Also add a point (**H**) at the bottom of the **vertical** line segment.

Your sketch should now look similar to this one:



- How many angles do you see?

- What kind of angles do you see?

6. Measure one of the angles using the Measure menu. Write the measure below.

7. Measure any other angle. What is its measure?

Is the perpendicular line you constructed also perpendicular to the parallel line or just to line **j**? Explain.

END OF ACTIVITY 10

• Activity 11 •

Swap jobs with your partner. Create a new sketch.

1. Refer to the diagram below to help you with this sketch.

Construct a line segment and a parallel line to the segment (just like Steps 1 and 2 in Activity 4).

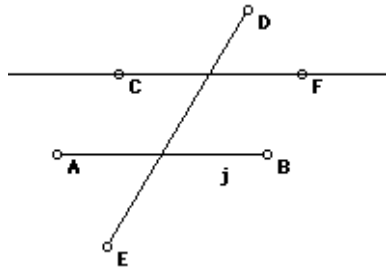
Place one point above the parallel line to the right of the midpoint of the line segment (like point **D** below).

Place another point below the line segment to the left of the midpoint of the line segment (like point **E**).

Select both points you just made and construct a segment. The segment should cross, or intersect both the first line segment and the parallel line.

Place a point on the parallel line to the right of the line segment (like

Your sketch should look similar to this one:



How many angles do you see?

What kind of angles do you see? (e.g., obtuse, acute, right, straight)

A line that passes through two or more other lines is called a **transversal**. A transversal that intersects two lines creates three different types of angle pairs.

1. Name a pair of corresponding angles.

2. Name a pair of vertical angles.

3. Name a pair of adjacent angles.

4. Measure a pair of corresponding angles (refer to page 6 for a reminder). What is their measure?

5. Measure a pair of vertical angles. What is their measure?

6. Measure a pair of adjacent angles and then add them. What is their sum?

7. Click on one endpoint of the transversal and drag it to the left or right.

What happens to the measures of the angles?

8. What do you notice about the measures of corresponding angles?

What do you notice about the measures of vertical angles?

What happens to the sum of the adjacent angles?

END OF ACTIVITY 11

Congruence

Activity 12

Swap jobs with your partner.

1. Open the file named **act9.gsp**.
2. Look at the angles in example 1.

What do you notice about the angles?

Are they **congruent**? Why or why not?

3. Look at example 2.

Name a pair of corresponding angles.

Are the two **angles** you just named congruent? Why or why not?

Are lines x and y congruent?

Are the two **triangles** congruent? Why or why not?

Click on one of the points of either triangle and drag to change the size of the triangle.

What do you notice about the other triangle?

3. Study example 3.

Are the two **polygons** congruent? Explain your answer.

Is XYS congruent to ABC ? Why or why not?

4. What do you think congruent means?

END OF ACTIVITY 12

Investigating Polygons

Polygons can be classified in a number of ways. They can be classified by the number of sides that make up the polygon. They can also be classified as regular or irregular. In the following activity, you will look construct and classify a number of polygons.

• Activity 13 •

Swap jobs with your partner. Create a new sketch.

How to draw a polygon

Use the line segment tool to click and draw segments connected at the vertices.

1. Construct the following polygons:

- triangle
- quadrilateral
- pentagon
- hexagon
- heptagon
- octagon

2. Write the number of sides that make up each polygon.

- triangle _____
 - quadrilateral _____
 - pentagon _____
 - hexagon _____
 - heptagon _____
 - octagon _____
-

3. Classify your polygons as **regular** or **irregular**.

- triangle _____
- quadrilateral _____
- pentagon _____
- hexagon _____
- heptagon _____
- octagon _____

END OF ACTIVITY 13

Classifying Triangles

Triangles can be classified by their angles and by their sides:

Classify by **angles**

Acute triangle ALL angles are acute

Obtuse triangle ONE angle is obtuse

Right triangle ONE angle is right

Classify by **sides**

Equilateral ALL sides are equal

Scalene NO sides are equal

Isosceles TWO sides are equal

Example:

A triangle with all acute angles and all equal sides would be classified as **acute and equilateral**.

• Activity 14 •

Swap jobs with your partner. Open the file named **act11.gsp**.

1. What do you notice about all of the sides?

What do the three angles add up to?

Are the angles ALL acute? How would you classify this triangle (check above or the Intro to Geometry program to check triangle classifications)?

_____ and _____

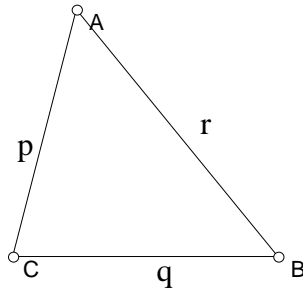
2. Click and drag point A **to the left** and make **ONLY** two of the sides congruent. Your sketch should look something like this:

$$\begin{aligned} r &= 1.60 \text{ inches} \\ q &= 1.30 \text{ inches} \\ p &= 1.30 \text{ inches} \end{aligned}$$

$$m\angle ACB = 76^\circ$$

$$m\angle ABC = 52^\circ$$

$$m\angle CAB = 76^\circ$$



3. What do you notice about the angles opposite the equal sides?

4. Are all the angles still acute?

How would you classify this triangle (check page 34 or the Intro to Geometry program to check triangle classifications)?

_____ and _____

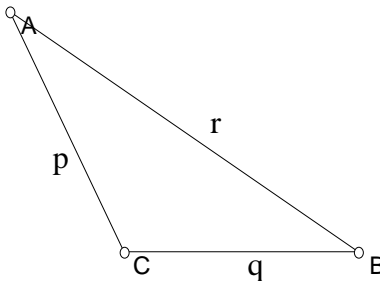
5. Click and drag point A to the left again until angle A is obtuse **BUT** keep still two sides congruent. The triangle might look something like this:

$$\begin{aligned} r &= 2.25 \text{ inches} \\ q &= 1.30 \text{ inches} \\ p &= 1.30 \text{ inches} \end{aligned}$$

$$m\angle ACB \approx 19^\circ$$

$$m\angle ABC \approx 30^\circ$$

$$m\angle CAB \approx 119^\circ$$



Measure angle C. Is it obtuse? Why?

How would you classify this triangle (check page 38 check triangle classifications)?

_____ and _____

6. Click and drag point A to the left until angle C is a **right** angle and all the sides are not equal.

Are all the sides **unequal**?

Can you classify this triangle?

_____ and _____

7. Draw a right isosceles triangle.
8. Draw an obtuse scalene triangle.
9. Draw an acute scalene triangle.

END OF ACTIVITY 14

Investigating Transformations

Translations. Follow the instructions in Activity 1 on the following page.

Open the file named Translation Tessellation for a fun sketch.

Reflections. Follow the instructions in Activity 2 on the following page.

Open the files named Coordinate Reflection, Glide Reflect, Half Head, More Reflection, Reflect, and Reflect Segment for some fun sketches.

Rotations. Follow the instructions in Activity 3 on the following pages.

Open the file named Rotate for a fun sketch.

Open other fun sketches named Symmetry and Twisted Triangles.
